A PRELIMINARY DESIGN AND LONGITUDINAL DYNAMICS STUDY OF A VTOL AIRCRAFT TO OPERATE EFFICIENTLY IN LOW ALTITUDE HIGH SPEED FLIGHT

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Symbols and Coefficients

\[ C_D \] \( \frac{D}{qS} \) aircraft drag coefficient

\[ C_L \] \( \frac{L}{qS} \) aircraft lift coefficient

\[ C_M \] \( \frac{M}{qS} \) aircraft pitching moment coefficient

\[ C_G \] aircraft center of gravity

\[ D \] aircraft drag

\[ F \] force

\[ GW \] aircraft gross weight

\[ I_{xx} \] moment of inertia about longitudinal (x) axis

\[ I = I_{yy} \] moment of inertia about pitching (y) axis

\[ I_{zz} \] moment of inertia about yawing (z) axis

\[ K_y \] radius of gyration about pitching axis

\[ L \] lift

\[ M \] mach number or moment about pitching axis as applicable

\[ \dot{M} \] air mass flow associated with engine inlets

\[ RPM \] engine revolutions per minute

\[ S \] aircraft wing area

\[ T \] engine thrust

\[ V \] aircraft velocity

\[ W \] weight

\[ X \] force in the x direction (positive forward)

\[ Z \] force in the z direction (positive downward)

\[ \bar{c} \] mean aerodynamic chord

\[ fps \] feet per second

\[ g \] gravitational force
SYMBOLS AND COEFFICIENTS (continued)

\( l_t \)  
distance CG to tail aerodynamic center

\( l_w \)  
distance CG to wing aerodynamic center

\( m \)  
aircraft mass

\( q \)  
dynamic pressure

\( u \)  
velocity in the \( x \) direction

\( w \)  
velocity in the \( z \) direction

Greek Symbols

\( \alpha \)  
angle of attack (angle between \( x \) axis and aircraft velocity)

\( \gamma \)  
angle between aircraft velocity and the horizontal

\( \phi \)  
angle between \( x \) axis and horizontal

\( \rho \)  
density of air

\( \lambda \)  
ratio of major to minor axis of an elliptical body or the root of a differential equation as applicable

\( \delta_c \)  
canard control surface deflection angle

\( \sigma \)  
engine thrust tilt angle from the longitudinal (\( x \)) axis

Subscripts

\( (\_)_c \)  
parameter associated with canard surface

\( (\_)_L \)  
parameter associated with lifting engines

\( (\_)_o \)  
a basic coefficient or an initial condition as applicable

\( (\_)_P \)  
parameter associated with the primary engines

\( (\_)_u \)  
\( \delta(\_)/\delta u \) change in parameter with a change in longitudinal velocity

\( (\_)_w \)  
\( \delta(\_)/\delta w \)

\( (\_)_\alpha \)  
\( \delta(\_)/\delta \alpha \)

\( (\_)_{\delta_c} \)  
\( \delta(\_)/\delta \delta_c \)

\( (\_)' \)  
\( \delta(\_)/\delta t \) change in parameter with time
Symbols and Coefficients (continued)

Computer Symbols

- Summing amplifier number 7 with two inputs
  A gain \( \frac{R_f}{R_1} \) of .5 for one input and of 1.0 for the other

- Integrating amplifier number 8 with one input
  A gain \( \frac{1}{RC} \) of .2
  An initial condition (IC) placed on the integrator

- Potentiometer number 20

- Servo Multiplier number 3, cup 2; with an input of
  +Y on one end of the cup and -Y on the other

- Diode

- Amplifier number 7 but dashed lines indicate that the
  inputs to this amplifier are shown in another part of circuit

Simulator Symbols

- Plug connector number 2 from Throttles

- Plug connector number 2 from Stick

- Plug connector number 2 from instruments

- Simulator panel instrument
SUMMARY

An investigation has been conducted to arrive at an aircraft configuration which combines vertical take-off and landing capabilities with an airframe that gives optimum performance in low altitude, transonic flight.

Initial consideration is given to the aerodynamic configuration which provides the most desirable lift, drag and stability characteristics in the transonic cruise range. Then, after considering the various engine types and arrangements that might be suitable for this requirement, a tentative aircraft configuration is chosen.

A longitudinal dynamic stability and control analysis of the tentative configuration is presented using a linearized small perturbation analysis, a computer small perturbation analysis and a flight simulator. The results of these analyses indicate that the original engine arrangement is unsatisfactory.

The turbojet engines in the original design are relocated, and the resulting design is found highly satisfactory via a simulator flight analysis. The final configuration is a high wing loading canard configured aircraft using lightweight lifting turbojet engines for vertical thrust and conventional turbojets on the wing tips for forward thrust.
INTRODUCTION

The current trend in the development of VTOL/STOL aircraft for use in the military fighter or attack mission is towards one airplane which can perform several different mission requirements. For example, the general operating requirement established for the current NATO VTOL Fighter requires suitable performance at high subsonic speeds at sea level, as well as supersonic speeds at high altitudes. As discussed later in this study, and as evidenced by the current interest in the use of variable sweep wings to allow an airplane to operate efficiently at both high and low altitudes, the blending of the various performance capabilities into one aircraft forces a compromise in both regimes.

The alternative to this multiple mission capability with the corresponding compromises in performance would be to tailor an aircraft for a specific mission with the result of several different aircraft for the several missions. Obviously, the development and procurement of several aircraft designs is more expensive than one aircraft to meet all requirements. However, the compromises required to perform this multiple mission by a single aircraft design may be so great as to justify the additional expense of several designs.

This study investigates the opportunities and problems associated with the aerodynamic design of an aircraft required to cruise in sea level transonic flight. Then, after a design considered suitable is decided upon, a detail analysis is made of the dynamic longitudinal stability and control characteristics.
PRELIMINARY DESIGN

In order to arrive at a suitable configuration for further dynamic study, it is necessary to review the design objectives and general design considerations for obtaining these objectives. Although the amount of design information required for a dynamic stability study of this type is small compared to a detailed design program, there are many items that must be considered before determining the range of values of the stability coefficients.

The aircraft configuration will be based upon a set of arbitrarily selected performance requirements and the handling qualities criteria of Reference (1). These criteria will be design goals and any necessary compromise of these requirements will be discussed when they occur.

The performance requirements are:

A. Vertical take off and landing from small prepared surfaces.

B. Cruising Mach Number approximately .9.

C. Cruising altitude approximately 1000 feet.

D. Cruising dynamic pressure = (.00230/2)(1020)^2 = 1200 PSF.

E. Fuselage in normal flight attitude in all modes of flight.

F. CW = 24000#/f

Cruising Flight Aerodynamic Considerations

When an aircraft is used for high speed, low level flight, the wings of the aircraft may detract from the performance capabilities that would otherwise be possible. A comparison of the drag polars of two different aircraft of the same fuselage planform area (Figure 1), one with a wing and the other wingless, provides an indication of the flight conditions in which a wing either helps or hinders the aircraft's performance. The wingless aircraft has a low profile drag coefficient (CpD) and a very high induced drag coefficient (dCD/dCL^2). In comparison, when wings are added
to the aircraft, a higher profile drag coefficient but a considerably lower induced drag result.

To obtain an optimum aircraft for specific flight conditions, it is necessary to consider on which side of the intersection of the two drag polars the aircraft will operate. For the transonic flight conditions, the lift coefficients will be very low because of the high dynamic pressure. Therefore, the use of the wingless aircraft would provide a lower drag (see $C_L$ for Mach .9 at sea level on Figure 1), and would be desirable if sufficient stability and control forces were available.

If the aerodynamic characteristics of the aircraft are to be designed for maximum cruise range at the low altitude transonic flight condition, the range equation for turbojet aircraft indicates the variables which have an effect on this range.

$$
R = 2 \left( \frac{C_L^{1/2}}{C_D} \right)^{1/2} \sqrt{\frac{391 W_0}{S \sigma}} \left[ 1 - \left( \frac{W_1}{W_0} \right)^{1/2} \right]
$$

where Range ($R$) is in miles, $c'$ is specific fuel consumption, $\sigma$ is the density ratio, $W_1$ is the final weight, and $W_o/S$ is the initial wing loading. The two parameters which are governed by the aerodynamic configuration are $(C_L^{1/2}/C_D)$ and the initial wing loading, $W_o/S$. Other terms held constant, an increase in either of these terms will create an improvement in range. However, since for level flight $W = C_L q S$ and the values of $W$ and $q$ are already fixed for the cruise condition, then the product $C_L S$ is fixed prior to the consideration of the range equation. Therefore:

$$
R = \frac{K}{C_D} \sqrt{\frac{C_L}{S}} = K \frac{C_L}{C_D} \sqrt{\frac{1}{C_L S}}
$$
but
\[ C_{L}S = \frac{W}{q} = \frac{24000}{1200} = 20 \]

or
\[ R = K\frac{C_{L}}{C_{D}} \]

So, to obtain maximum range for this given condition, a configuration should be selected which provides a maximum L/D ratio for the design lift coefficient.

To see if this is a practical consideration, the plots of \( C_{L} \) and L/D
vs angle of attack for several bodies of varying cross section shown in
Figure 2 may be studied. This Figure shows the lift-drag relationship for
the bodies which all have the same cross sectional area but varying ratio
of the major to minor axis (\( \lambda \)) of an elliptical cross section. These bodies
are considered because they would have the minimum basic drag coefficient of
any possible configuration. Based upon the reference area used in Figure 2,
the required \( C_{L} \) at cruise may be calculated:

\[ C_{L_{\text{req}}} = \frac{W}{qS} = \frac{20}{(72)^2} = .00385 \]

It is seen that at this \( C_{L} \) the elliptical cross section with \( \lambda = 3 \) would pro-
vide the maximum L/D ratio. This cross section at the 5 degree angle of
attack which provides the \( C_{L} = .00385 \) would give an L/D of 5.

Although this may not appear to be a high L/D ratio when compared to
cruising L/D ratios of current fighter aircraft (L/D > 12), it must be com-
pared with the L/D ratios of these aircraft when flying transonically at
sea level. At this speed and altitude the L/D ratios for the F4H/F104 air-
craft are around 1.2.
For stability and control reasons it is impossible to fly the elliptical cross section body alone and certainly control surfaces must be added. However, this discussion indicates that the maximum range of an aircraft required to fly at low altitude in high speed flight may be increased as much as three times by optimizing the aerodynamic design around this condition.

Static Longitudinal Stability and Control Considerations

In addition to providing a VTOL capability, the use of power plants for lifting as well as forward thrust provides the opportunity for designing the aircraft's aerodynamic configuration for the cruise flight condition alone. The aerodynamic forces for hovering and low speed flight are negligible; therefore either reaction or some form of engine thrust control will be required to stabilize and control the aircraft under low speed conditions. A problem is to decide when, in the velocity profile, the control augmentation is to give way to the aerodynamic control. In general, the lower the speed for which aerodynamic control is required, the larger the lifting and control surfaces must be to provide this control. But the larger these surfaces are, the higher the drag in cruise conditions and thus a compromise of cruise design is required. The major reason for requiring as low an aerodynamic control speed as possible is to provide safety of aerodynamic control in case of engine failures over as much of the flight range as possible. Since the aircraft considered herein is viewed with respect to military mission requirements and most engine failures of conventional single engine jet aircraft result in pilot ejection, it is believed that low drag, aerodynamically stable, easily controlled cruise flight should be a prime objective even though adequate control by aerodynamic surfaces will not occur until a relatively high speed is reached.
Conventional aircraft when advancing from low subsonic to transonic speeds frequently encounter performance and control problems as a result of significant changes in static stability characteristics (see Figure 3). The primary longitudinal problem is the increased longitudinal stability which results from the increased static margin. This increased static margin is usually caused by a rearward shift of the center of pressure of the wing, the loss of wing downwash at the tail, and the stabilizing influence of the wing lift carried over to the fuselage afterbody. Although this effect is not dangerous, it does depreciate the aircraft's performance in the following ways:

1) The increase in stability creates a need for additional nose up trim, increasing the drag and reducing the lift on the horizontal stabilizer. This reduction of lift necessitates trimming the aircraft to a higher angle of attack. These effects all act to reduce the lift/drag ratio in transonic flight.

2) This stability increase requires a larger elevator deflection for a given maneuver. In addition the large elevator deflection required for trimmed flight reduces the amount of deflection available for maneuvering flight. Thus the stability increase decreases the maneuvering capabilities of the aircraft.

The two main analytical factors affecting this longitudinal stability increase are $C_{M0}$ and $dC_M/dC_L$. Factors which would reduce the stability increase would be to make $C_{M0}$ increase positively with Mach number and to make $dC_M/dC_L$ less negative. Several methods have been discussed to do this; e.g., fuselage camber to increase $C_{M0}$ as the fuselage center of pressure moved aft or auxiliary canard surfaces which could be extended as $dC_M/dC_L$ increases to provide a destabilizing moment.
These problems are alleviated somewhat by the ability of the VTOL aircraft to have poor stability or be neutrally stable throughout the low speed (control augmentation) range, and therefore to locate the center of gravity position for cruise conditions.

The following paragraphs discuss the advantages and disadvantages of several different aerodynamic configurations for providing low drag and suitable stability in the transonic range.

Specific Aerodynamic Configurations

Canard Configuration:

The canard configuration (see above sketch) has certain stability and control advantages, at transonic and supersonic speeds, over the conventional wing-afterbody-tail aircraft arrangement. The canard configuration static stability increases much less when advancing into transonic speeds than does the conventional aircraft (see Figure 3). This is partially due to the elimination of afterbody and horizontal tail so that the lift carryover effects of the wing on the afterbody and the downwash changes at the tail are avoided. Since the change in stability with increasing Mach number is small, the static margin may be kept small, thus minimizing control movements and deflections.
for trimming. The control effectiveness of a canard benefits from a long moment arm, allowing small deflections to provide adequate control moments. With only small deflections, the induced drag and lift changes due to control deflections are kept small.

The major problem of the canard configuration is the stability and control in low speed flight. At the higher angles of attack the control deflection for a nose up moment adds to the body angle of attack placing the canard surface at the stall angle with a modest body angle of attack. Thus it is difficult to get large positive moments and to trim at maximum lift for low speed flight. This is a serious deficiency for conventional aircraft because of the landing velocity requirement. However, for the flight profile required herein, this may not be a deficiency at all since $C_L \neq C_{L_{\text{max}}}$ may never be required.

One problem area that is readily apparent in the use of this configuration is the pressure variation across any jet engine intake duct which is aft of the canard control surfaces. The effect of the canard vortices may eliminate the possibility of satisfactory engine performance. However, little data are available to determine how detrimental these effects may be and, therefore, for the purposes of this study, these effects will not be considered in finalizing the aerodynamic configuration.

Wingless Configuration:
The stability changes with Mach number can also be minimized by the use of a wingless airplane. (Two possible configurations are shown in the above sketches). As in the canard configuration, there is no wing lift afterbody effect or downwash over the horizontal tail. Also the small horizontal tail surface adds little to the over-all center of pressure shift as its center of pressure shifts with Mach number.

The effect of fuselage cross section on the static longitudinal and lateral stability characteristics has been investigated in Reference 4. Figure 4 indicates a comparison of certain longitudinal stability characteristics for a circular fuselage-45 degree swept horizontal tail configuration versus a relatively flat fuselage-45 degree swept horizontal tail configuration. The flattening of the fuselage has the following effects on the longitudinal stability:

1) An increase in the slope of the lift curve ($C_{Lx}$).
2) An increase in drag with angle of attack, because this flattened body acts as a very low aspect ratio wing.
3) An increase in the positive destabilizing pitching moment ($C_{Mx}$).
4) An increase in the lift/drag ratio at the higher angles of attack.

As a result of this study, it is seen that the aircraft fuselage cross section would depend strongly upon the specific range of Mach numbers under consideration. The lower Mach number, the higher the $C_L$ required, and thus the more flattened body for a given angle of attack. However, this flatter body would have a more positive pitching moment slope, necessitating a larger horizontal stabilizing surface and consequently more profile drag, with less efficient operation. Therefore, the narrower the cruise speed range, the more efficient is the optimum design.
A low speed wind tunnel investigation of a wingless jet VTOL model with swept horizontal and vertical stabilizers has been reported in Reference 5. A sketch of the design considered and a plot of some of the results are given in Figure 5. The results of this investigation were that satisfactory static longitudinal and lateral stability could be achieved with this configuration. In addition, it was pointed out that, due to the low slope of the lift curve for this type configuration, the accelerations created by gusts at low altitudes would be less than in a conventional configuration.

Low Aspect Ratio Wing with Wing Tip Located Jets:

A less radical departure from conventional aircraft designs is indicated in the above sketches. The suitability of rotating the jet engines is discussed elsewhere in this study. However, the location of these engines results in certain advantages and disadvantages which will be discussed here.

Aerodynamically, these configurations would produce more drag at the low lift coefficient cruising speeds; but this performance degradation may be a necessary compromise to achieve the other certain advantages of this design. The center of pressure movement and resulting increase in longitudinal stability as these designs pass into the transonic flight regime would be considerably greater than the canard or wingless designs previously discussed.
Two significant advantages in these designs would occur in the rolling characteristics. Reference 6 discusses the problem of roll inertia coupling. In the designs of aircraft for higher speed, the trend is to concentrate more and more of the mass along the aircraft's longitudinal axis, resulting in a much larger moment of inertia about the pitch axis \(I_{yy}\) than about either the roll axis \(I_{xx}\) or the yaw axis \(I_{zz}\). Under this condition, the maximum roll rate of the aircraft will be limited by the build up of centrifugal forces about the rolling axis (displaced \(\alpha\) in pitch from the \(XX\) axis). When these centrifugal forces exceed the aerodynamic stability forces, the aircraft pitches up uncontrollably and catastrophe usually results. Placing the jet engines on the wing tips increases \(I_{xx}\) and reduces the problems of inertia coupling. The other advantage of this design in rolling is increase in aileron power due to more aileron area and a longer moment arm.

Locating the engines on both the wing tips and tail tips affords the use of differential power for hovering control about all three axes, as discussed in Reference 7.

The relative size of the tail versus the wing of the design having both wing and tail tip engines deserves some comment. For hovering flight, the center of gravity of the aircraft must coincide with the center of thrust of the engines. For static stability, the center of pressure of the aerodynamic surfaces must be aft of the center of gravity. From these two considerations the problem is to design an engine arrangement with the center of thrust on the forward portion of the aircraft and an aerodynamic surface arrangement providing a center of pressure aft of this center of thrust. As a typical example of the effects of these requirements an arrangement providing a forward center of thrust with 2/3 of the thrust on the wing tips and 1/3 of
the thrust on the tail tips was investigated. Calculations were made to
determine the ratio of tail area to wing area necessary to place the result-
ant center of pressure behind the center of gravity. The results of these
calculations indicated that the tail area would have to be at least 75%
greater than the wing area for static stability. The results of these cal-
culations are interpreted to mean that, for any reasonable distribution of
thrust between the wing tips and tail tips, the tail area must be very large
and, in general, would be larger than the wing area.

Engine Considerations

To achieve flight over the transonic speed range, the turbojet or turbo-
fan engine is the readily apparent requirement. The propeller (ducted or not)
loses almost all efficiency in the transonic regime, while the ramjet achieves
satisfactory efficiencies only at speeds in excess of Mach 1.5.

The ratio of cruise to take-off thrust required presents an engine
matching problem. The thrust required by current high performance air-
craft for horizontal flight and maneuvering is usually about 30% of the
aircraft take-off weight. If all engines are used for both forward thrust
and vertical lift, being as efficient either way, then the engines which are
designed to provide total thrust in excess of aircraft weight must be throttled
back greatly or some engines must be shut down. Since the specific fuel con-
sumption of a conventional turbine engine goes up considerably as the RPM is
reduced below the design range (usually 85-95%), the reduction of power on
each engine to 1/3 maximum is almost out of the question. To alleviate this
some, the use of afterburners could be considered. An afterburner usually
augments the thrust at 100% RPM by about 50%. Thus an engine could operate
efficiently at about 50% maximum afterburner power. There are several
serious objections to using afterburning jet engines for vertical take-off. The afterburner greatly increases the temperature of the exhaust gases and these gases impinging directly on a surface would require undesirably extensive landing pad preparation. The accurate control over afterburner thrust variations that would be required in vertical flight has not yet been accomplished. Also, the afterburner approximately doubles the length of a turbojet engine. Therefore, the rotating jet using an afterburner would be undesirably long (104 inches for the J35-5 afterburning engine with 3850 pounds thrust). If engines are to be shut down during cruising flight, their design and location should be such as to create as little drag as possible while shut down.

Several different engine configurations have been taken through various stages of development. The various types that are applicable to the performance requirements herein are discussed in the following paragraphs. Since the use of jet engines provides no inherent stability during very low speed flight, the use of jet reaction controls is assumed for all models discussed.

Lift Fan Engine:
The lift fan engine (see above sketch) diverts the turbine exhaust air from a conventional turbojet engine into the tip turbines of a large lift fan for vertical thrust. Transition from hovering to horizontal flight with this engine is accomplished by a close coordination of the tilting of louvered vanes at the fan exit and the repositioning of the diverter valve in the turbojet. Transition is complete when the turbojet is acting conventionally and the fan openings are closed.

Recent tests on this type engine at NASA Ames Research Center have indicated that a 76 inch diameter fan coupled to a GE-J85-5 engine produces 7430# lifting thrust and 2580# horizontal thrust for an installation weight of 1145#. Proponents of this system anticipate achieving Thrust/Weight ratios of 10/1 by coupling the fan to an engine also used for forward thrust, and to achieve Thrust/Weight ratios of 15/1 by using turbojets designed for lifting only.

Advantages of this system are:

1) High mass-low velocity and temperature air provided by the fan is easier on landing areas than hot-high velocity turbojet exhausts.

2) The horizontal/vertical thrust ratio is about the required amount for comparison with current high performance aircraft.

3) Several fans could be coupled to several engines so that failure of one engine would not cause catastrophic failure.

Large disadvantage of the system is:

1) The fan is large and occupies volume in a fuselage installation.
Lift Engines:

If a turbojet engine is designed for only short period, low altitude, low speed operation, a considerable weight reduction for a given thrust is possible. Rolls-Royce is currently developing a series of lifting engines and has achieved thrust/weight ratios in excess of 10/1. Proponents of this system feel it is well within the state of the art to develop systems with a 20/1 thrust/weight ratio.

Because the cruise flight thrust required is only 30 per cent of the lifting thrust required and the thrust/weight ratios for lifting engines would be about two times as large as the cruising engines, a blend of engines designed solely for thrust at low speed and engines designed for cruise flight is feasible. The Short SC-1 VTOL research aircraft (see Figure 6A) has successfully used a combination of lifting engines plus one engine providing only horizontal thrust. The lifting engines in this aircraft may be tilted plus or minus 30 degrees in the X-Z plane to provide initial horizontal acceleration forces and to aid in deceleration. Less than one minute operation is required of the lift engines in either take-off or landing transitions. The aircraft uses reaction controls until the aerodynamic controls are fully effective and all lift is taken by the wings at about 160 knots. Major advantage of this engine over the General Electric lift fan engine is the smaller amount of aircraft planform occupied for a given thrust. The corresponding disadvantage is the higher exhaust gas temperature and velocity.

Rotating Jet Engine:

Rotating the entire turbojet engine for vertical thrust was tried with some success with the Bell Air Test Vehicle (Reference 8 and Figure 6B) in 1955.
Since that time little has been accomplished along these lines. Pilots could perform 360 degree turns with this test vehicle in the hovering stage but longitudinal and directional control was below a satisfactory level. Only one marginally satisfactory transition at altitude was accomplished before the completion of this program.

The advantage of this method is that full thrust would be available for both hovering and horizontal flight. However, the problems associated with the mechanical rotation of the engine, the gyroscopic coupling, the center of gravity movement with engine rotation, and the variations of drag coefficient with engine angle of attack appear much more formidable than those associated with thrust diversion.

Diverted Thrust Engine:

The thrust from a horizontally mounted turbojet may be redirected from horizontal to a wide range of angles by diverting the high pressure air immediately aft of the turbine.

The Bell X-14 (Figure 6C) has successfully employed the principle for VTOL research operation. The thrust diverter consists of a vane cascade, which turns the exhaust gases from vertical to horizontal as the airplane transitions from hovering to horizontal flight.
The Bristol Siddeley BS-53 engine to be used in the Hawker P.1127 VTOL (see above sketch) diverts its thrust by swiveling the exhaust of both forward fan air and the aft turbine exhaust air. No test results of this engine are available yet.

If the diverted thrust engine is developed to the stage that there is very little thrust loss in the diversion, and the mechanical equipment to perform this diversion is reliable and light, this method would have a large advantage over the rotating jet principle. This method would avoid the engine alignment, rotating fuel control, and gyroscopic effects of tilting the jet engine.

### Dynamic Longitudinal Stability Considerations

As a result of the foregoing considerations, the optimum aerodynamic configuration would be of either a wingless or canard design. It is now necessary to consider the dynamic characteristics of these configurations.

Reference 9 presents the damping in pitch characteristics of several tailless swept wing body combinations. The results of this study indicated the following:

1) All models were statically stable throughout the Mach number range investigated (M_{0.85} to 1.30).

2) The rotational damping in pitch derivatives \( (C_{\theta} + C_{\alpha}) \) of all models were very small and either negative (stable) or positive (unstable).

3) Since the damping in pitch derivatives were low, the total damping factor consisted mainly of the contribution of the slope of the lift curve.

Reference 10 presents the damping in pitch of tailless delta wing body combinations. The results of this study indicate the following:
1) All models were statically stable throughout the Mach number range investigated but were dynamically unstable, to various degrees, at transonic speeds.

2) The total damping factor as well as the rotational damping in pitch derivatives were extremely small at subsonic and supersonic speeds and were unstable at transonic speeds.

Reference 11 presents the damping in pitch characteristics for a canard missile configuration. The results of this study indicate the following:

1) The model was statically stable throughout the range investigated (M .9 to 1.25) over the angle of attack range ± 6 degrees.

2) The effect of Mach number changes on static and dynamic stability was very small. The maximum shift in aerodynamic center was 12 percent of the mean aerodynamic chord.

**Preliminary Design Conclusions**

Based upon this brief analysis of the general factors affecting the design of an aircraft to cruise in low altitude transonic flight conditions, the choice is narrowed to a wingless design limited by poor dynamic stability and control characteristics or a canard design which may introduce problems of pressure variation over the engine inlets due to canard vortices.

Because of the outstanding stability and control characteristics of the canard configuration in the desired cruise range, this design is selected for additional detailed study. Other than to point out the canard influence on engine inlet conditions, this interference effect will not be considered because of the sparseness of experimental data available on the subject.

The dynamic longitudinal stability and control characteristics of the configuration detailed in Figure 7 is chosen for further study. At cruise
flight conditions this configuration will provide an (L/D) of 4.8. This compares with an (L/D) of about 2 or less for a current fighter aircraft flying transonically at sea level. Therefore, this design could provide two to three times the range in the sea level high speed flight regime because of its optimization for this condition. This aircraft will be powered by three lifting engines ahead of the CG of the aircraft and one primary deflected exhaust engine at the rear.
Detailed Configuration

The aircraft to be studied in the longitudinal dynamics analysis is shown in Figure 7. This aircraft consists of a triangular wing and canard control surface, both of aspect ratio 2, and a modified Sears-Haack body. The primary powerplant is an aft mounted turbojet whose intakes are on the side of the aircraft at the wing roots and whose exhaust cone may be directed through any angle from directly aft (0 degrees) to 30 degrees forward of straight down (120 degrees). To augment the primary powerplant thrust for hovering and low speed flight, lifting engines are provided. These lifting turbojet engines consist of a battery of three lightweight engines which may be tilted, as a unit, to the angles of plus or minus 30 degrees from vertical (60 degrees - 120 degrees). These lifting engines are located ahead of the aircraft CG so as to balance the moments created by the deflected primary engine thrust.

The dimensions and weight characteristics are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (L)</td>
<td>50 feet</td>
</tr>
<tr>
<td>Wing Area (S)</td>
<td>237 square feet</td>
</tr>
<tr>
<td>Weight (W)</td>
<td>24,000 pounds</td>
</tr>
<tr>
<td>Center of gravity (CG)</td>
<td>.04 c</td>
</tr>
<tr>
<td>Moment of inertia (I_y)</td>
<td>74,500 slug-feet squared</td>
</tr>
<tr>
<td>Radius of gyration (K_y)</td>
<td>10 feet</td>
</tr>
<tr>
<td>Mean aerodynamic chord (\bar{c})</td>
<td>14.5 feet</td>
</tr>
</tbody>
</table>
The engine locations and maximum ratings are as follows:

Lifting engines

Max. thrust ($T_L_{\text{max}}$) 16,500 pounds
Exhaust to CG ($X_{L_e}$) 10 feet
Intake to CG ($X_{L_i}$) 10 feet

Primary engine

Max. thrust ($T_P_{\text{max}}$) 11,000 pounds
Exhaust to CG ($X_{P_e}$) 15 feet
Intake to CG ($X_{P_i}$) 0

Aerodynamic Characteristics:

Most of the stability derivatives for this study have been taken from Reference 12. This Reference presents wind tunnel results for a model similar to the one under study for the speed range of M.70 to M.2.22. The data available for M.70 has been corrected by a .9 correction factor for compressibility effects. This value was arrived at as a compromise between the two dimensional flow correction factor and the less effective corrections of the slender wing theory.

The aerodynamic derivatives taken from this Reference are as follows:

- $C_{1\alpha} = 2.58$/radian
- $C_{m\alpha} = -0.322$/radian
- $C_d = 0.0126 + 2.52\alpha^2$
- $C_{m\delta_c} = 0.30$/radian

These values are plotted against angle of attack in Figure 8. This Figure shows the results of Reference 12 and the comparison with the above analytical values.
The aerodynamic damping derivatives must be estimated by analytical means. The contribution of the rate of change of angle of attack to the damping of the aircraft is very small for this configuration and will be considered as included in the pitch rate damping derivative \( C_{m\dot{\alpha}} \). This derivative is estimated as follows:

Contribution of canard

\[
\frac{\partial C_M}{\partial \dot{y}} = - C_{L\alpha_c} \frac{S_c}{\alpha} \frac{\ell_t}{c} \frac{1}{u}
\]

Contribution of wing

\[
\frac{\partial C_M}{\partial \dot{y}} = - C_{L\alpha} \frac{\ell_y}{c} \frac{1}{u}
\]

\[
\begin{align*}
u_{CM_{\dot{y}}} &= \frac{-(.193)(26)}{14.5} - \frac{2.58(6.68)}{14.5} = -1.54
\end{align*}
\]

Therefore to include effects of \( C_{M\alpha} \) use:

\[
\begin{align*}
u_{CM_{\dot{y}}} &= -2.0
\end{align*}
\]

**Engine Forces Acting on the Aircraft**

In addition to the conventional aerodynamic forces acting on this aircraft design, there are the strong inlet momentum forces associated with the engines. Normally the thrust associated with a turbojet engine is the "net" thrust, which is the sum of the "gross thrust" and the "ram drag" terms which have vectors in the same line. Because there is a large angle between these vectors for the aircraft under consideration, the "net" thrust must be separated into its components.

The "ram drag" forces are equal to the change of momentum of the air entering the engine diffuser. Therefore this would equal the mass flow of the air times the velocity of the air, with the force aligned in the direction of flight. The "gross thrust" is equal to the mass flow of air plus fuel at the exit times the velocity at exit aligned in the direction of thrust.
Figure 9 is a plot of typical turbojet engine variations showing the relationship of mass flow (\(\dot{m}\)) and gross thrust (T) versus engine RPM and aircraft velocity. By using these typical variations and the simple momentum theory, approximations may be made for the forces associated with the engines in this aircraft.

Lifting Engines. Since the lifting engines are submerged in the fuselage, there will be no ram pressure, i.e., there will be no increase in mass flow or thrust with velocity as shown in Figure 9. In fact, it appears that, due to reduced pressure on the upper surface of the fuselage, there may be a reduction of air mass flow with velocity. Due to the sparseness of data, both theoretical and experimental, for analyzing this flow, and since this factor is small compared to the effect of RPM, it is felt that the best available approximation is to neglect the ram effects of the lifting engines.

Therefore, based upon the typical engine variations and the geometry shown in the following sketch, the forces and moments associated with the lifting engines are:
\[
T_L = \frac{T_L(\text{max})}{1 \times 10^6} \text{ RPM}^3 = 0.0165 \text{ RPM}_L^3 \quad \text{(lbs)}
\]

\[
\dot{M}_L = \frac{T_L(\text{max})}{7200} \frac{3.85 \text{ RPM}_L}{100} = 0.0832 \text{ RPM}_L \quad \text{(slugs)}
\]

\[
F_X(\text{engines}) = T_L \cos \sigma_L - \dot{M}_L V \cos \alpha
\]

\[
F_Z(\text{engines}) = -T_L \sin \sigma_L - \dot{M}_L V \sin \alpha
\]

\[
M(\text{engine}) = 10 T_L \sin \sigma_L + 10 \dot{M}_L V \sin \alpha + 2 \dot{M}_L V \cos \alpha
\]

The moment arm of ten feet is to the center of the lift engine group. The application of the momentum force two feet above the center of the engines is arrived at by considering it at the lip of the engine intake.

Primary Engine. Since the intake for the primary engine is approximately perpendicular to the velocity the ram pressure will have full effect in this engine. It is assumed that the inlet momentum force is applied at the CG of the aircraft.

Based upon the following sketch, the forces and moments associated with the primary engine are:
\[ T_p = \frac{T_p(\text{max. static}) \, \text{RPM}_p^3}{1 \times 10^6} \left[ 1 + 0.29 \times 10^{-3} \nu \right] = 0.011 \, \text{RPM}_p^3 \left[ 1 + 0.29 \times 10^{-3} \nu \right] \]

\[ \dot{M}_p = \frac{T_p(\text{max. static}) \, \text{RPM}_p \, (3.85)}{7200} \left[ 1 + 0.18 \times 10^{-3} \nu \right] \]

\[ = 0.059 \, \text{RPM}_p \left[ 1 + 0.18 \times 10^{-3} \nu \right] \]

\[ F_{X_p} = T_p \cos \sigma_p - \dot{M}_p V \cos \alpha \]

\[ F_{Z_p} = -T_p \sin \sigma_p - \dot{M}_p V \sin \alpha \]

\[ M_p = -15 \, T_p \sin \sigma_p \]

**Control Forces**

At low velocities the aerodynamic control forces are negligible and some auxiliary control force is required. For this longitudinal control a small thrust variation in \( T_L \) will be used. Reference 1 requires that control power in hovering be sufficient to produce an angular displacement of at least \( \frac{45}{(W + 1000)^{1/3}} \) degrees at the end of one second of full control displacement.

\[ \theta_1 = \frac{45}{(25000)^{1/3}} = 1.54^\circ = \frac{\hat{\theta}^2}{2} \]

\[ M_{req} = 10 \dot{\theta}_{req} = 74500 \, \frac{3.09}{57.3} = 4030 \text{ ft. lbs.} \]

\[ \Delta T_L_{req} = \frac{4030}{10} = 403^\# \]

Since 500\# represents only 1\% variation in RPM at 100\%, this is a very small percentage variation in \( T_L \).

Let full stick travel give a \( \pm 403^\# \) variation in \( T_L \) and a \( \pm .3 \) radian variation in canard angle of attack with respect to fuselage center line.

Then the moment change due to thrust may be combined with the canard moment.
\[ L_M = C_{M_{cc}} \delta_c qS \delta + \frac{\delta_c}{\frac{4030}{3}} \]
\[ = \delta_c \left[ \frac{(0.3)(0.023)(237)(14.5)}{2} V^2 + 13400 \right] \]
\[ = \delta_c \left[ 1.20 V^2 + 13400 \right] \]
\[ M_{\tau\tau} = \delta_c \left[ 1.61 \times 10^{-5} V^2 + 0.18 \right] \]

The Flight Profile

By referring to the above sketch, the controls required and the procedures to be used during takeoff and transition may be discussed. The controls required are based upon the premise that manual pilot control throughout transition is mandatory.

Takeoff, hover and climb is accomplished by using the engine thrust for both lifting forces and longitudinal pitching control. Since the primary engine creates a nose down pitching moment about the CG and the lifting engines create a nose up pitching moment about the CG, and these thrust forces are much greater than any aerodynamic forces during this portion of the flight, a close moment balance must be maintained. This moment balance is accomplished in the early part of the flight by changing the tilt of the primary and lifting engines by the same amount, and at the same time having the throttles for each engine linked together to provide equal percentage thrust changes (equal RPM).
Thus, when the primary engine thrust vector is between plus or minus 30 degrees from the vertical, the lifting engine tilt will be exactly the same angle. Also by mechanically linking the throttles so that the signalled RPM is the same for both sets of engines, any change in throttle setting would cause a change in the thrust of each set of engines in a proportionate amount. As discussed earlier in this paper, a 1% change in engine RPM should create sufficient moments for control of the aircraft in the hovering flight. Therefore, if a change in thrust of the lifting engines accompanies longitudinal stick movement, control is provided for this flight regime. Also, if the longitudinal trim of the aircraft is accomplished by repositioning the center position of the stick, longitudinal trim is provided when the engine forces are the primary control as well as at higher speeds when the canard surface is the primary control.

The transition from hovering to conventional forward flight is accomplished with the aircraft attitude approximately horizontal and with the angle of attack always below the stalling angle of attack. During the first portion of the transition, only the engine thrust forces are available to support the aircraft. The speed is increased by tilting the engines forward a few degrees to provide forward thrust. As the speed increases, the aerodynamic lift and moment forces become increasingly important. The aircraft is accelerated to the velocity at which the aerodynamic lift can sustain the weight. At this point the lifting engine tilt would be at its forward limit with an identical tilt of the primary thrust ($\sigma_F = \sigma_L = 60^\circ$). The aircraft angle of attack would be low because the thrust of the engines would still be providing considerable lift. This speed transition can be accomplished by any of several various combinations of engine RPM, engine tilt ($\sigma$), and aircraft pitch angle ($\theta$). This is due to the gradual transition from engine thrust to aerodynamics as the
primary means of lift and control. Figure 10 shows the various combinations that provide horizontal steady, accelerating and decelerating performance. The method used for the calculation of this chart is given in Appendix I.

Upon reaching the speed at which the aerodynamic lift will support the weight, the lifting engines may be shut down. Several procedures could be considered to accomplish this. The moments created by the engines are still powerful and any reduction in the moment of the lifting engines would have to be closely coordinated with reduction of moment created by the primary engine. At the completion of shutting down the lifting engines, the primary engine thrust vector should be pointed aft while the lifting engines are at their forward tilt limit of 60°. One way to accomplish this could be to reduce all engines to idle so that moment contribution by the engines would be small and then rotate the primary engine and secure the lifting engines. However, this procedure would cause a large deceleration of the aircraft during the maneuver and be objectionable. Another way to accomplish this maneuver is to change the thrust direction of the primary engine such that its reduction in nose down moment is equal to the reduction in nose up moment due to lifting engine thrust changes. As the results of the simulator study show later, this procedure is not as difficult as it might seem and appears to be satisfactory. Upon completion of this maneuver the aircraft is in conventional forward flight.

The procedure for transitioning from conventional flight back to hovering flight is accomplished by reversing the above steps.

**Longitudinal Force Equations**

Two sets of axes systems are used in this report. The small perturbation analysis uses the stability axis system (axes fixed in the body but initial position parallel and perpendicular to the relative wind) for all speeds.
except hovering. The hovering small perturbation analysis and the analog computer simulation uses a body axis system with the X axis along the fuselage center line. Referring to the following sketch the resulting force equations are:

![Diagram of aircraft with labeled axes and forces](image)

**Stability Axes Oriented**

\[
\begin{align*}
\sum F_X &= 0 = T_P \cos (\sigma_P + \alpha) + T_L \cos (\sigma_L + \alpha) - (\dot{M}_L + \dot{M}_P) V \\
&\quad - C_{DqS} - W \sin (\theta - \alpha) - M \dot{\theta} \\
\sum F_Z &= 0 = - T_P \sin (\sigma_P + \alpha) - T_L \sin (\sigma_L + \alpha) - C_{LqS} + W \cos (\theta - \alpha) \\
&\quad + mV(\dot{\phi} - \dot{\alpha}) \\
\sum M &= 0 = 10 T_L \sin (\sigma_L) - 15 T_P \sin (\sigma_P) + 10 \dot{M}_L V \sin \alpha + 2 \dot{M}_L V \cos \alpha \\
&\quad + \left[ C_{M\alpha} \alpha + C_{MC} \bar{c}_c \right] qS \bar{c} - mK_y \dot{\theta}.
\end{align*}
\]
Body Axes Oriented

Note: \( \dot{x} = u, \dot{z} = w \)

\[ F_x = 0 = T_p \cos \sigma_p + T_L \cos \sigma_L - (\dot{M}_L + \dot{M}_P) u - C_D q \sin \alpha \]

\[ + C_L q \sin \alpha - mg \sin \phi - m(\dot{u} + \dot{\omega}) \]

\[ F_z = 0 = -T_p \sin \sigma_p - T_L \sin \sigma_L - (\dot{M}_L + \dot{M}_P) w - C_D q \cos \alpha \]

\[ - C_L q \cos \alpha + mg \cos \phi - m(\dot{\omega} - \dot{\phi}u) \]

\[ M = 0 = 10T_L \sin \sigma_L - 15T_p \sin \sigma_p + 2\dot{M}_L u + 10 \dot{M}_L w \]

\[ + \left[ C_{M\alpha} \alpha + C_{M_C} \delta_C \right] q \delta_C^2 - mK_{Y \phi} \phi \]

Small Perturbation Analysis - General Comments

The longitudinal equations of motion for this aircraft are nonlinear simultaneous differential equations and as such have no closed solution. Two of the methods available for the study of the dynamics of these equations are the "small perturbation analysis" and a nonlinear analysis by analog computer simulation. Both of these methods are used herein in order to compare results and to determine the validity of the small disturbance analysis for this type aircraft. Also the simulation allows a pilot opinion evaluation of the aircraft's flight characteristics.

The derivation of the equations of motion for an aircraft are available in any comprehensive text on the subject (see References 13 and 14) and will not be repeated here. The important features to be considered here are the assumptions made and the application of the results. The assumptions which are common to the complete analysis and a discussion of their validity are as follows:

1) The airplane is a rigid body with the XZ plane a plane of mirror symmetry.

2) The changes of moments of inertia are negligible.

3) The elevator free modes need not be considered because the elevator in this aircraft would be power boost controlled.
4) There are no rotor gyroscopic effects. Although it is probable that there will be some gyroscopic coupling in this type aircraft, it is assumed that this coupling will be minimized by contrarotating rotors and therefore this effect may be neglected for this analysis.

These assumptions allow the longitudinal and lateral modes of the aircraft to be uncoupled with the resulting arrangement of three equations in three unknowns.

**Linearized Small Perturbation Analysis**

Using the procedures given in Reference 14, calculations were made to determine the effect of small moment perturbations on the aircraft while flying at speeds of 0, 100, 200, 300 and 400 feet per second. Additional assumptions used for this analysis are that all angle perturbations are small and the velocity perturbations from the referenced steady state velocity are small. The calculations made to obtain the resulting dynamics are given in Appendix II.

The results of this linear analysis are plotted on the complex plane in Figure 11.

**Analog Computer Small Perturbation Analysis**

The small perturbation response of this aircraft was also studied with the use of the Goodyear Electronic Data Analyzer (GEDA L3). This computer facility provided a total of 24 operational amplifiers and 5 servo multipliers. (See Figure 21). Because of the availability of the servo multipliers, certain nonlinear functions could be included in the computer program. The detailed equations of the computer programming and the assumptions used are given in Appendix III. In general, the small angle relationships of the linear perturbation analysis were applied, but the velocities were allowed to vary throughout the entire flight range. In addition the engine forces were treated as nonlinear functions.
These equations were first placed in the computer in an arrangement that could be used to check computer results against the results of the small perturbation analysis. This computer program is shown in Figure 12. In this program, each of the engine force inputs and the elevator input were placed on the potentiometers as shown on the left hand side of the diagram. Initial conditions of u, w, and $\theta$ were placed in the circuit at their respective integrators. The rest of the connections were made as they would be after the simulator was connected into the circuit. Therefore, by selecting a proper combination of parameters from the transition chart (Figure 10) for steady state flight, and introducing these parameters in as initial conditions in the computer program, a pitching force input could be used to study the response of the aircraft at these conditions.

Runs were made in this manner at the 0, 100, 200, 300 and 400 feet per second conditions with the initial parameters identical with those of the linearized small perturbation analysis. In addition, runs were made for the initial pitch angle zero ($\theta = 0$) at the 300 and 200 fps speeds. The results of these runs are shown in Figures 14 through 20. It is noted that these results are based upon small angle assumptions as in the previous analysis but that the velocity was allowed to make large variations and its nonlinear effects were incorporated. In addition a parabolic drag polar was used instead of the straight line relationship in the linear analysis. The programmed potentiometer settings for each run are shown in Figure 13.

A comparison of these results with the linearized analysis shows good agreement in the short period response characteristics at the various speeds. There is also good agreement on the phugoid period, however the damping is somewhat inconsistent. This inconsistency in the damping results is considered...
uninformative, however, because the phugoid period involved is long (> 50 seconds) and all times to 1/2 or 2 amplitude are greater than one period. This appears to be so long that it will not affect the handling qualities of the aircraft.  

**Resulting Longitudinal Dynamics**

The results of this pre-simulator analysis may then be summarized. Reference may be made to Figure 11, the complex plane representation of the resulting dynamics. In hovering, the predominant mode of motion will be a straight divergence with a time to double amplitude on the order of 2 - 3 seconds. Obviously the only way to control this highly undesirable result, if it can be controlled at all, is to have a rapid and powerful control force. The simulator study indicates the requirements for this control.

In early transition the engine forces will be predominant over the aerodynamic forces and as the aerodynamic forces become stronger with increasing speed, this divergence will approach a neutral stability (150 - 200 fps) and become a very long period, lightly damped phugoid motion. Meanwhile, the short period mode of motion which was unnoticed in very low speed flight while the divergence was predominant, will become the important handling quality mode. Between 200 and 400 fps the period of this mode will go from 9 seconds to 4.5 seconds as the speed increases. The cycles to 1/2 amplitude is about one in all cases.

So, as could be expected, since this aircraft is optimized around high dynamic pressure conditions the dynamic characteristics at the lower speeds are very poor compared to conventional aircraft. Desirable characteristics would not be reached until the velocity is above 400 fps. Therefore it is important to know whether these dynamics are safely controllable throughout the transition, if the transition may safely be aborted at any point in the
transition, if an autopilot is a mandatory requirement for this operation, and if sufficient control power could be made available. The simulator study has been made to provide some answers for these problems.
Simulator Description

The cockpit simulator shown in Figures 21 and 22 was constructed to allow a pilot opinion evaluation of the resulting aircraft dynamics and control characteristics. The simulator was simply constructed but provided control forces and instrument panel readings sufficient to represent the variables in the longitudinal motions of the aircraft needed for pilot control.

The force inputs were introduced through the throttle quadrant and the control stick. The throttle quadrant utilized two throttles, one for the lifting engines and one for the primary engine. The throttle movements were made proportional to engine RPM. Potentiometers were arranged on each throttle to provide cubic functions for the thrust vs RPM relationships and linear functions for the air mass flow vs RPM relationships. These two throttles were equipped to be mechanically linked together for the period of flight in which the engine moment forces and the engine RPM's were to be balanced and equal. They could be mechanically separated when, in the last phase of transition, the lifting engines were to be shut down. A three position spring center switch was available on one throttle to change the tilt angle ($\sigma$) of both engines. The control stick which had fore and aft movement only was linked to a torsion bar upon which was mounted two linear potentiometers for the introduction of elevator force inputs and engine or reaction control forces. The center position of this torsion bar could be changed to relieve trim pressures by a three position spring center switch on the stick which controlled a positioning motor attached to the torsion bar.
The instrument panel consisted of a group of dc microammeters arranged and labeled to provide necessary pilot information for flight. By referring to the numbered instruments in the following sketch the information presented was:

![Instrument Panel Diagram]

The Simulator Instrument Panel

(1) and (2) The engine RPM's, $\text{RPM}_L$ and $\text{RPM}_P$ respectively. Graduated from 0 to 100%.

(3) The engine tilt angle ($\sigma$) with 0 degrees centered at the top and plus or minus 100 degrees from vertical available on either side.

(4) The aircraft altimeter ($h$) graduated from 0 to 5000 feet altitude.

(5) The aircraft air speed indicator ($u$) graduated from 0 to 500 feet per second.

(6) The aircraft pitch angle ($\phi$) with 0 center and plus or minus 25 degrees available.

(7) The elevator or canard deflection angle with the same graduation as $\phi$.

(8) The rate of climb indicator graduated from 0 to plus or minus 50 feet per second (3000 fpm).

Except for the stick trim motor, this simulator was designed to be operated by DC voltages from the analog computer and banana plug connections were on all terminals for direct connection into the computer patch panel.
The Simulator Program

The simulator was connected into the previously discussed patch panel program used for the small perturbation analysis. As was mentioned before, the program for the small perturbation analysis was arranged to accommodate the simulator with as few changes as possible. The resulting introduction of the simulator into the previous program is shown in Figure 23.

The Simulator Flight Analysis

The first steps in flying the simulator were to place initial conditions in the computer to represent flight at 300 fps. To again check the validity of the over-all simulation, a small perturbation in the elevator was made and the dynamic response recorded. This response is shown in Figure 24. Comparison of this response with the small perturbation analysis response of Figure 19 shows good agreement.

Flights were then made by always starting at the 300 fps initial condition, which was stable, and attempting to fly into the higher speed range to complete the lifting engine shutdown and back into the lower speed range to accomplish hovering.

After some practice, the forward transition could be accomplished. The best procedure was found to be to increase speed to 400 fps and then to establish a constant rate of change of the primary engine exhaust angle ($\sigma_p$). Then the reduction of the lifting engine thrust could be manually coordinated to control the moment balance. The reverse procedure was used to restart the lifting engines. A recording of the important parameters during this maneuver is given in Figure 25. It is noted from the recording that, although the transition could be accomplished, large pitching deviations were encountered whenever the coordination of change of lifting engine thrust with the primary exhaust deflection was not precise. It was concluded that this maneuver should normally be automatically controlled,
but could be safely controlled manually if the automatic control should fail.

Starting at 300 fps and slowing the airplane to the hovering condition could not be accomplished. As the airplane slowed, the dynamic and control responses became progressively worse. The stick movements required to maintain controlled flight became progressively larger until at about 100 fps the available control was not enough. The recording shown in Figure 26, which was one of the best controlled flights, indicates this progressive increase in control requirements. The amount of engine thrust control available was increased by 2 and 4 times in other flights to see if hovering could be accomplished, without success.
Redesign Considerations

Since emergency control by the human pilot in case of autopilot failure is a requirement of this type aircraft, ways must be found to provide suitable stability for control. An investigation of the modes of motion of the aircraft in hovering reveals that there is a large instability due to the $M_u$ and $M_w$ terms. By referring to the following sketch, we can see qualitatively how these destabilizing forces are created in hovering;

As the velocity ($u$) increases the ram pressure on the inlet of the primary engine increases giving a resulting increase in thrust ($T_p$). Since this thrust is directed downward, the resulting moment change about the CG is nose down. In a similar way the thrust of the lifting engines is a function of the vertical velocity ($w$). An increase in $w$ causes a change in both the inlet momentum forces and the thrust ($T_L$). However, the nose up moment caused by the inlet momentum forces is greater than the nose down moment caused by the reduction in thrust. Therefore, the net result is a nose up moment with increase in vertical velocity $w$.

The complex plane may be used to demonstrate how these forces affect the resulting modes of motion. Looking again at the stability determinant for the hovering condition and using symbols instead of the quantitative values as used in Appendix II:
\[
\begin{bmatrix}
(X_u - d) & 0 & -g \\
Z_u & Z_w - d & 0 \\
M_u & M_w & M_v d - d^2 \\
\end{bmatrix}
\begin{bmatrix}
x \\
w \\
\theta \\
\end{bmatrix}
= 0
\]

\[(X_u - d)(Z_w - d)(M_v d - d^2) - Z_u g(M_w) + M_u g (Z_w - d) = 0\]

By first allowing \(M_w\) to vary while holding \(M_u\) zero and then vice versa, we see how the roots move in response to certain variations.

First setting \(M_u\) equal to zero and plotting the resulting locus of roots as \(M_w\) varies:

\[
\frac{Z_u g}{d(X_u - d)(Z_w - d)(M_v - d)} = 1
\]

\[
M_w = -
\]

\[
M_w = +
\]

Showing a plus value of \(M_w\) as is in the aircraft being studied creates a static instability or a real divergence. However, a minus value of \(M_w\) would cause first a static stability then a stable oscillation and then an unstable oscillation as the value is increased.

Now setting \(M_w\) equal to zero and plotting the resulting locus of roots of \(M_u\):
\[ \frac{M_u\bar{g}}{d(X_u - d)(M_c - d)} = 1 \]

Again showing the sign of \( M_u \) which is in this aircraft (-) creates a static instability, while the opposite sign would provide an unstable oscillation.

It may be observed that if the signs of these derivatives could be reduced and/or reversed, the hovering dynamics of this aircraft would approach those of a helicopter, which has an unstable oscillation and a real convergence. This unstable oscillation is controllable although an autopilot is usually required for precise hovering control.

Since the real divergence of the configuration being studied is uncontrollable, via the simulator study at speeds below 100 fps, it is apparent that a great deal of change needs to be made in the design to achieve hovering stability. Therefore, the engine type and locations will be changed in the simulator to try to provide adequate pilot control throughout hovering and transition.
Modified Design

The engine types and locations in the previous design were based upon the assumption that a deflected exhaust engine could be combined with lifting engines in an efficient manner so that the forward thrust engine could also carry some of the weight in hovering flight. The results of this analysis show that, even if the deflected exhaust principle could be developed to provide the wide range of thrust angles required, the use of this type engine in the manner proposed is prohibited by the dynamic stability and control characteristics. Therefore, the engine arrangement will be changed as indicated in the following paragraphs.

Lift Engines:

The lifting engines will now provide a total thrust in excess of take off weight. These engines will be positioned in the aircraft at an angle perpendicular to the fuselage center line and will not be allowed to rotate. These engines will be located about the aircraft center of gravity. Enough thrust in excess of weight will be provided for climb requirements and to make trim changes for various center of gravity positions.

Primary Engines:

The thrust vector of the primary engines will be fixed and aligned with the fuselage center line. Since this thrust direction is now fixed, the engines may be located on the wing tips to place the engine intakes out of the influence of the canard trailing vortices.

This modified design is shown in Figure 27.

A discussion of the force equations and the longitudinal dynamics in hovering for this redesign is given in Appendix IV. A comparison of the hovering roots of this design with those of the previous design and of the
S-55 helicopter is presented in Figure 23. It is seen that this design will provide roots of the same form but of smaller magnitude than those of the S-55.

**Flight Profile of Modified Design**

Take off, hovering and climb for this design is accomplished on the lifting engines alone. The lift is provided by the engine thrust and the pitching control is provided by reaction controls at the extreme nose and tail. The high pressure air for the reaction control is bleed air from the lifting engine compressors.

Transition is initiated by either lowering the nose to provide a forward vector from the lift engines or by starting the forward thrust engines. Further acceleration is provided by merely increasing the RPM of the forward thrust engines. Since the engines in this design do not give the large moments of the previous design, there are no difficult engine coordination requirements as before. As the speed increases the lifting engines may be shut down either gradually as the lift increases or more rapidly when the velocity is large enough so that the aerodynamic lift can equal the weight. The combinations of aircraft pitch angle, engine thrusts and velocity that may give steady trimmed flight are so varied that a transition chart, similar to Figure 10, cannot be constructed for this aircraft. However, the simulator study discussed next indicates that the procedures for transition are easily varied and thus the detailed procedures for an aircraft of this type would probably be determined by other considerations than those discussed here.

**Simulator Analysis of Modified Design**

Changes were made in the simulator program to represent the redesigned aircraft. The revised simulator connections are shown in Figure 29. As before, perturbation runs were made at various speeds to compare the dynamics with the other configuration. These runs at 0, 60, 200 and 300 fps are shown in Figures 30 through 33.
The results of these runs show that the short period frequency is increased about 20% and the phugoid damping is improved. It also verifies the long period divergent oscillation in hovering.

Next a series of simulator flights were made to study the aircraft characteristics throughout the flight range. The aircraft was found to be easily controllable in hovering. Highly satisfactory flights were made starting at zero velocity and altitude and proceeding to climb to 1000 feet altitude followed by transition to forward flight powered by the forward thrust engines alone. The transition could be aborted at any speed during transition and safely returned to hovering.

Figure 34 shows a typical flight from hovering at zero altitude to 100 fps at 1000 feet and then a return to initial conditions. This flight was conducted on the lifting engines alone. Figure 35 shows a rapid acceleration and deceleration flight from hovering at sea level to a completely transitioned condition at 500 fps at 1000 feet and then back to hovering at sea level. Figure 36 shows a complete flight from hovering at sea level to 500 fps at 1000 feet with trimmed flight conditions at several discrete airspeeds during transition. This Figure also shows the return transition in a similar manner.

The results of these simulator flights indicate that the modified design has highly satisfactory longitudinal dynamic stability and control characteristics.
CONCLUSIONS

A study has been conducted to arrive at an aircraft configuration that is optimized around a particular operational requirement. This requirement consists of a VTOL capability and maximum range in transonic low altitude flight.

The conclusions drawn from this study are as follows:

1) The optimum aerodynamic configuration for this operational requirement is a high wing loading canard configured aircraft. The high wing loading is required for satisfactory cruise range performance in the high dynamic pressure flight condition. The canard configuration is the best for providing suitable longitudinal stability and control throughout the flight regime.

2) The optimum engine arrangement consists of a battery of high thrust/weight turbojet engines designed for lifting and transition only and additional engines used for forward thrust only. The possibility of using the forward thrust engines for both lifting and forward thrust by deflecting the exhaust was thoroughly studied and ruled unsatisfactory because of uncontrollable hovering and low speed longitudinal dynamics.

3) The resulting configuration as shown in Figure 27, would provide a low altitude transonic range capability for a given fuel consumption of about 2-1/2 times that of current fighter aircraft flying in this condition. This configuration would possess suitable longitudinal dynamics and control capabilities throughout its flight range. Although it would be desirable to have automatic pilot control of the aircraft, it could be safely flown by a human pilot from hovering condition to the maximum speed considered.

4) The longitudinal control characteristics specified in Reference 1 are sufficient to control this aircraft.
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15. Gillis, C.L. and Vitale, J.A.: Wing on and wing off longitudinal characteristics of an airplane configuration having a thin unswept wing of aspect ratio 3, as obtained from rocket propelled models at Mach numbers from 0.8 to 1.4. NASA TN D-7, 1960.
APPENDIX I

THE TRANSITION CHART

By substituting the appropriate aerodynamic and thrust forces in the longitudinal force equations, the various combinations of engine thrust, engine tilt angle and aircraft pitch angle for steady level flight at discrete air speeds may be calculated.

For example let $V_o = 300$ fps and $\alpha = 60^\circ$. Substituting these values into the stability axes equations (page 29): 

\[
\begin{align*}
F_x &= 0 = 0.011 \text{ RPM}_L^3 \left[1 + 0.29 \times 10^{-3}(300)\right] \cos (60 + \alpha) + 0.0155 \text{ RPM}_L^3 \cos (60 + \alpha) \\
&- 0.0322 \text{ RPM}_L (300) - 0.059 \text{ RPM}_L \left[1 + 0.18 \times 10^{-3}(300)\right] (300) - C_D qS \\
F_z &= 0 = 0.011 \text{ RPM}_L^3 \left[1 + 0.29 \times 10^{-3}(300)\right] \sin (60 + \alpha) + 0.0155 \text{ RPM}_L^3 \sin (60 + \alpha) \\
&- C_L qS + W 
\end{align*}
\]

Assuming the moments are balanced and $\text{RPM}_p = \text{RPM}_L$

\[
\begin{align*}
F_x &= 0 = 0.0285 \text{ RPM}_L^3 \cos (60 + \alpha) - 45.2 \text{ RPM} - 24500 C_D \\
F_z &= 0 = 0.0285 \text{ RPM}_L^3 \sin (60 + \alpha) - 24500 C_L + 24000 
\end{align*}
\]

A simultaneous solution of these nonlinear equations by trial and error yields:

\[
\begin{align*}
\alpha &= 0.157 = 9^\circ \\
\text{RPM} &= 81^\circ 
\end{align*}
\]

The results of many calculations of this type may be plotted as the transition chart of Figure 10. This chart has velocity as the abcissa and $\theta + \psi$, (the engine tilt angle from horizontal) as the ordinate. Combinations required for level steady flight and combinations providing accelerating or decelerating level flight are depicted. For example, at the 300 fps velocity previously considered, steady flight may be attained using the $\sigma = 60^\circ$, $\psi = 9^\circ$, RPM = 81°.
combination or the \( \phi = 79^\circ, \varepsilon = 0 \), RPM = 95\% combination or any combination in between giving level flight. Combination providing level flight with 

\[ (\phi + \varepsilon) > 79^\circ \]

give decelerations while combinations providing level flight with 

\[ (\phi + \varepsilon) > 69^\circ \]
give accelerations. Also the "lift engine shutdown regime" is shown. This shows the velocity vs angle of attack combination for which the aerodynamic lift is equal to the aircraft weight \( (\phi = 60^\circ) \). Note that only at velocities above 375 fps can the lift forces support the weight.
Linearized Small Perturbation Analysis in Hovering

Using the body axes system of notation and dividing the force equations by the airplane mass and the moment equation by the airplane moment of inertia, the longitudinal equations of motion may be defined in matrix form as follows (see Reference 14):

\[
\begin{bmatrix}
X_u - d & X_v & -g \cos \theta_o - w_o d & u \\
Z_u & Z_v - d & -g \sin \theta_o + u_o d & x \\
M_u & M_v & M_d - d^2 & \theta
\end{bmatrix}
\begin{bmatrix}
u \\
w \\
\theta
\end{bmatrix}
= 0
\]

where

\[
d = \frac{d}{dt} ( )
\]

\[
X_u = \frac{1}{m} \left( \frac{\partial X}{\partial u} \right) \text{ engine and aerodynamic terms}
\]

\[
M_u = \frac{1}{I_Y} \left( \frac{\partial M}{\partial u} \right) \text{ engine and aerodynamic terms}
\]

The forces associated with hovering equilibrium may be calculated as follows:

\[
u_o = w_o = 0 \quad \sigma_p = \sigma_L = 90^\circ \quad \delta_o = 0 \quad \text{RPM}_p = \text{RPM}_L
\]

\[
\sum F_z = 0 = -T_p - T_L + mg = -0.0275 \text{ RPM}^3 + 24000
\]

\[
\text{RPM}^3 = 8.73 \times 10^5
\]

\[
\text{RPM} = 95.5\%
\]
Then the matrix coefficients may be evaluated

\[ X_u = \frac{1}{m} (\dot{\mathbf{M}}_L - \dot{\mathbf{M}}_T) = -\frac{1472 \text{ RPM}}{745} = -0.0189 \]

\[ X_w = 0 \]

\[ X_\zeta = 0 \]

\[ \dot{z}_u = -\frac{3 \text{TP}}{\delta u} \frac{1}{m} = -\frac{0.111 \text{ RPM}^3 (0.29 \times 10^{-3})}{745} = -3.75 \times 10^{-4} \]

\[ \dot{z}_w = -\frac{(\dot{\mathbf{M}}_L + \dot{\mathbf{M}}_T)}{m} = -\frac{1472 \text{ RPM}}{745} = -0.0189 \]

\[ E_\zeta = 0 \]

\[ M_{u} = \frac{-15}{I_y} \frac{\partial \text{TP}}{\partial u} \frac{1}{I_y} + \frac{2 \dot{\mathbf{M}}_L}{I} = \frac{-15}{7.45} \frac{0.011 \text{ RPM}^3 (0.29 \times 10^{-3}) + 0.1734 \text{ RPM}}{7.45 \times 10^{-4}} = -3.29 \times 10^{-4} \]

\[ M_{w} = \frac{10}{I} \frac{\partial \text{TP}}{\partial w} + 10 \frac{\dot{\mathbf{M}}_L}{I} = \frac{10}{7.45} \frac{0.0189 (\text{RPM}^3 (0.29 \times 10^{-3})) + 0.0852 \text{ RPM}}{7.45 \times 10^{-4}} = 5.71 \times 10^{-4} \]

\[ M_{\zeta} = -M_w = -5.71 \times 10^{-4} \]

Note that the ram effects on the lifting engines are considered here in deriving \( M_w \) and \( M_\zeta \). They are considered only in this hovering analysis where the effects, though small in magnitude, could contribute to the resulting dynamics because the other dynamic forces are also very small and few. The result of these effects do contribute very little to the dynamics even in hovering and therefore the neglecting of these effects in the forward flight regime is appropriate.

The resulting calculations are:

\[
\begin{bmatrix}
0.0189 + \dot{d} & 0 & 32.2 \\
0.00375 & 0.0189 + \dot{d} & 0 \\
0.000329 & -0.00371 & 0.000571 \dot{d} + \dot{d}^2
\end{bmatrix}
= 0
\]

\[
\begin{bmatrix}
u \\
x \\
\zeta
\end{bmatrix}
\]
with the resulting characteristic roots:

\[
\lambda = -0.12 + j 0.18 \\
\lambda = -0.025 \\
\lambda = +0.22
\]

**Linearized Small Perturbation Analysis in Forward Flight**

Using the stability axis system of notation and dividing the force equations by the airplane mass and the moment equation by the airplane moment of inertia, the longitudinal equations of motion may be defined in matrix form as follows (see Reference 14).

\[
\begin{bmatrix}
X_u - d & X_\alpha & -g \cos \gamma_0 \\
L_u & L_\alpha - u_0 d & -g \sin \gamma_0 + u_0 d \\
M_u & M_\alpha & M_\alpha d - d^2
\end{bmatrix}
\begin{bmatrix}
X \\
\alpha \\
\gamma_0
\end{bmatrix}
= 0
\]

Using as an example the steady state condition of \( u_0 = 300 \), \( \alpha_0 = 0.157 \), \( \text{RPM} = 81\% \), the matrix coefficients may be calculated as follows:

\[
mX_u = \frac{\partial \tau_p}{\partial u} \cos (\sigma_p + \alpha) + \frac{\partial T_L}{\partial u} \cos (\sigma_L + \alpha) - \frac{\partial C_D}{\partial u} qS - \frac{\partial q}{\partial \alpha} C_D S \\
- (\dot{M}_P + \dot{M}_L) - \frac{\partial \dot{M}_P}{\partial u} u \\
\frac{\partial \tau_p}{\partial u} \cos (\sigma_p + \alpha) - C_D \rho VS - (\dot{M}_P + \dot{M}_L) - \frac{\partial \dot{M}_P}{\partial u} u \\
\frac{1}{m} \frac{\partial \tau_p}{\partial u} \cos (\sigma_p + \alpha) = \frac{0.11 \text{ RPM}^3}{745} (0.29 \times 10^{-3}) \cos 69^\circ = 8.19 \times 10^{-4} \\
\frac{1}{m} C_D \rho V_o S = \frac{0.075 (0.00230)(300)(237)}{745} = 0.0164 \\
\frac{\dot{M}_P + \dot{M}_L}{m} = \frac{0.0882 \text{ RPM}_P + 0.059 \text{ RPM}_L}{745} = 0.0160 \\
\frac{1}{M} \frac{\partial \dot{M}_P}{\partial u} u_0 = \frac{(0.059)(\text{RPM})(0.18 \times 10^{-3})300}{745} = 3.47 \times 10^{-4} \\
X_u = +0.082 \times 10^{-2} - 1.64 \times 10^{-2} - 1.60 \times 10^{-2} - 0.035 \times 10^{-2} = -0.032
\]
\[ M\alpha = -T_p \sin (\phi_p + \alpha) - T_L \sin (\phi_L + \alpha) - C_{D\alpha} qS \]

\[ \frac{T_p}{M} \sin (\phi_p + \alpha) = \frac{.011 \text{ (RPM)}^3 \left[1 + .29 \times 10^{-3} (300)\right]}{745} \sin (69) = 7.95 \]

\[ \frac{T_L}{M} \sin (\phi_L + \alpha) = \frac{.0165 \text{ (RPM)}^3 \sin 69}{745} = 11.0 \]

\[ \frac{C_{D\alpha} qS}{M} = \frac{(1.02)(.00230)(300)^2 237}{2 (745)^2} = 33.6 \]

\[ X\alpha = -7.95 - 11.0 - 33.6 = -52.55 \]

\[ X_\theta = 0 \]

\[ mZ_u = -\frac{\partial T_p}{\partial u} \sin (\phi_p + \alpha) - \frac{\partial T_L}{\partial u} \sin (\phi_L + \alpha) - \frac{\partial C_{L\alpha}}{\partial u} qS - C_L \frac{\partial q}{\partial u} S \]

\[ 1 \frac{\partial T_p}{\partial u} \sin (\phi_p + \alpha) = \frac{.011 \text{ (RPM)}^3 (.29 \times 10^{-3}) \sin 69}{745} = 2.12 \times 10^{-3} \]

\[ 1 \frac{\partial T_L}{\partial u} \sin (\phi_L + \alpha) = \frac{.405 (.00230)(237)(300)}{745} = 89 \times 10^{-3} \]

\[ Z_u = -2.12 \times 10^{-3} - 89 \times 10^{-3} = -91 \times 10^{-3} = -0.091 \]

\[ MZ\alpha = -T_p \cos (\phi_p + \alpha) - T_L \cos (\phi_L + \alpha) - C_{L\alpha} qS \]

\[ \frac{T_p}{M} \cos (\phi_p + \alpha) = \frac{(7.95) \cos (69)}{\sin 69} = 2.91 \]

\[ \frac{T_L}{M} \cos (\phi_L + \alpha) = \frac{11.0 \cos 69}{\sin 69} = 4.04 \]

\[ \frac{C_{L\alpha} qS}{M} = \frac{2.58}{1.02} (33.6) = 85.0 \]

\[ Z\alpha = -2.91 - 4.04 - 85.0 = -92 \]

\[ Z_\theta = 0 \]
\[
I_y M_u = -15 \frac{\partial T_p}{\partial u} \sin \sigma_p + 10 \frac{\partial T_L}{\partial u} \sin \sigma_L + 10 \dot{M}_L \sin \alpha + 2\dot{M}_L \cos \alpha \\
+ \left[ C_{M\alpha} \alpha + C_{M\delta_c} \delta_c \right] \frac{\partial \alpha}{\partial u} \\
+ \frac{15}{I_y} \frac{\partial T_p}{\partial u} \sin \sigma_p = \frac{(15)(.011)(RPM)^3}{7.45 \times 10^4} \times 300 \times 10^{-3} \sin(60) = 2.95 \times 10^{-4} \\
\frac{\partial T_L}{\partial u} = 0 \\
+ \frac{10 M_L \sin \alpha}{I_y} = \frac{10 \times (.0882)(RPM) \sin(9)}{7.45 \times 10^4} = 1.69 \times 10^{-4} \\
+ \frac{2 M_L \cos \alpha}{I_y} = \frac{2 \times (.0882)(RPM) \cos(9)}{7.45 \times 10^4} = 1.83 \times 10^{-4} \\

To find \left[ C_{M\alpha} + C_{M\delta_c} \delta_c \right] \delta_c \text{ must be computed from the moment equation. In a conventional aircraft this term would be zero, but due to the}\]
\[
\Delta T_L \text{ variance with stick in this aircraft it is not zero.} \\

Assuming RPM_p = RPM_L except for the stick input the moment equation becomes:
\[
M = 0 = -15(.011)(RPM_p)^3(.29 \times 10^{-3})(300) \sin(60) \\
+ 10(.0882)(RPM) 300 \sin(9) + 2(.0882)(RPM) 300 \cos(9) \\
+ \left[ C_{M\alpha} \alpha \frac{\partial \sigma}{\partial c} + I_y \delta_c (1.61 \times 10^{-5} u^2 + .180) \right] \\
= -6590 + 3350 + 4220 \\
+ (-.322)(.157)(24500)(237)(14.5) + I_y \delta_c \left[ 1.45 + .180 \right] \\
= -17020 + 1.63 I_y \delta_c \\
\delta_c = .140 \\

Then:
\[
\left[ C_{M\alpha} + C_{M\delta_c} \right] \delta_c = (-.322)(.157) + .30(.140) = -.0085 \\
- \frac{.0085 \delta_c}{I_y} = -\frac{.0085(0.00230)(237)(300)(14.5)}{7.45 \times 10^4} = -2.70 \times 10^{-4} \\
M_u = -2.95 \times 10^{-4} + 1.69 \times 10^{-4} + 1.83 \times 10^{-4} - 2.70 \times 10^{-4} = -2.13 \times 10^{-4} 
\]
\[ I_y M_{\alpha} = 10 M_{LV} \cos \alpha - 2 M_{LV} \sin \alpha + C_{M_\alpha} q \hat{S} \]
\[
\frac{10 M_{LV} \cos \alpha}{I_y} = \frac{10(0.0882)(\text{RPM})(300) \cos (9)}{7.45 \times 10^4} = 0.283
\]
\[
\frac{2 M_{LV} \sin \alpha}{I_y} = \frac{0.283 \sin (9)}{(9) \cos (9)} = 0.009
\]
\[
\frac{C_{M_\alpha} q \hat{S}}{I_y} = \frac{(-0.322)(24500)(237)(14.5)}{7.45 \times 10^7} = -1.54
\]
\[ M_{\alpha} = 0.283 - 0.009 - 1.54 = -1.27 \]
\[ M_\theta = 0 \]
\[ I_y M_{\phi} = C_{M_\phi} q \hat{S} = \frac{\rho}{2} u \hat{S} = \frac{2(0.00230)(300)(237)(14.5)}{2} \]
\[ M_{\phi} = -2.38 \times 10^2 \frac{7.45 \times 10^4}{10^4} = -0.0319 \]

The resulting characteristic matrix is:
\[
\begin{bmatrix}
0.032 + \alpha & 52.6 & 32.2 \\
0.091 & 92 + 300 \alpha & -300 \alpha \\
2.13 \times 10^{-4} & 1.27 & 0.0319 \alpha + \alpha^2
\end{bmatrix}
\]

With the characteristic roots
\[
\lambda = -0.174 \pm j1.11
\]
\[ \lambda = -0.0080 \pm j0.074 \]

Phugoid
\[ P = \frac{2\pi}{\omega_n} = \frac{2\pi}{0.074} = 85 \text{ sec} \]
\[ T_{1/2} = \frac{0.693}{0.0080} = 87 \text{ sec} \]

Short Period
\[ P = \frac{2\pi}{1.11} = 5.67 \]
\[ T_{1/2} = \frac{0.693}{0.174} = 4 \]
Similar calculations have been made at 100 fps and 200 fps. For the steady state condition of \( u_0 = 100, \alpha = 11^\circ, \sigma = 75^\circ, \text{RPM} = 93.5\%, \sigma_c = .105 \) the characteristic determinant is:

\[
\begin{vmatrix}
0.0251 + d & 34.3 & 32.2 & u \\
0.0397 & 11.56 + 100d & -100d & x \\
4.13 \times 10^{-4} & .066 & .0106d + d^2 & \sigma
\end{vmatrix}
\]

yielding the modes of motion:

\[
\begin{align*}
\lambda &= .18 & T_2 &= 3.8 \\
\lambda &= -.03 & T_{1/2} &= 23 \\
\lambda &= -.15 \pm j.35 & T_{1/2} &= 4.6 & P = 18 \text{ sec}
\end{align*}
\]

For the steady state condition of \( u_0 = 200, \alpha = 10^\circ, \sigma = 69.4^\circ, \text{RPM} = 88.4\%, \sigma_c = .151 \) the characteristic determinant is:

\[
\begin{vmatrix}
.03 + d & 40.6 & 32.2 & u \\
.0683 & 42.5 + 200d & -200d & x \\
2.57 \times 10^{-4} & .492 & .0213d + d^2 & \sigma
\end{vmatrix}
\]

yielding the modes of motion:

\[
\begin{align*}
\lambda &= +.006 \pm j.085 & P = \frac{2\pi}{0.065} &= 74 \text{ sec} & T_2 &= \frac{.693}{.006} = 115 \text{ sec} \\
\lambda &= .14 \pm j.68 & P = \frac{2\pi}{.68} &= 9.2 \text{ sec} & T_{1/2} &= \frac{.693}{.14} = 5.0
\end{align*}
\]

For the steady state condition of \( u_0 = 400 \text{ fps}, \alpha = 11.6^\circ, \sigma = 0, \text{RPM}_L = 0, \text{RPM}_p = 85\% \sigma_c = .215 \) the characteristic determinant is:

\[
\begin{vmatrix}
.037 + d & 62 & 32.2 & u \\
.152 & 160 + 400d & -400d & x \\
0 & 2.74 & .0426d + d^2 & \sigma
\end{vmatrix}
\]
yielding the modes of motion:

\[ \lambda = -0.18 \pm j0.11 \quad P = \frac{2\pi}{1.11} = 57 \quad T_{1/2} = \frac{693}{615} = 3.8 \]

\[ \lambda = -0.222 \pm j1.64 \quad P = \frac{2\pi}{1.34} = 4 \quad T_{1/2} = \frac{693}{222} = 3.1 \]
APPENDIX III

DEVELOPMENT OF COMPUTER PROGRAM EQUATIONS

The body axis system is used for the computer study. The resulting force equations for this system of notation as developed in equations on page 30 will be repeated here for continuity. These force equations are:

\[ F_x = 0 = T_p \cos \alpha_p + T_L \cos \alpha_L - (M_L + M_p) u - C_D qS \cos \alpha + C_L qS \sin \alpha - mg \sin \theta - M(u + \dot{\omega}) \]

\[ F_z = 0 = T_p \sin \alpha_p - T_L \sin \alpha_L - (M_L + M_p) w - C_D qS \sin \alpha - C_L qS \cos \alpha + mg \cos \theta - M(\dot{w} - \dot{\omega}u) \]

\[ M = 0 = 10 T_L \sin \alpha_L - 15 T_p \sin \alpha_p + 2 M_L u + 10 \dot{M}_L \dot{w} \]

\[ + \left[ C_{Mx} \alpha + C_{Mx2} \right] qS^2 \frac{\dot{\theta}}{\dot{\omega}^2} - mK^2 \dot{\omega} \]

Then if the following small angle assumptions are made:

\[ \sin \alpha \approx \alpha = \tan^{-1} \frac{w}{u} \approx \frac{w}{\sqrt{u^2 + w^2}} \]

\[ V = \frac{u}{\cos \alpha} = u \]

\[ R/c = V \sin \gamma = u \left( \frac{w}{u} - \frac{w}{u} \right) = \omega - w \]

\[ \sin \theta = \theta, \cos \theta = 1 \]

The following terms may be simplified as indicated:

\[ C_D(\cos \alpha) v^2 \approx (C_{D_0} + C_{D_\alpha} \alpha^2) u^2 \approx C_{D_0} u^2 + C_{D_\alpha} u^2 w^2 \]

\[ C_L(\sin \alpha) v^2 \approx C_{L_\alpha} \alpha \frac{w}{u} u^2 \approx C_{L_\alpha} u^2 \]

\[ C_L(\cos \alpha) v^2 \approx C_{L_\alpha} \alpha u^2 \approx C_{L_\alpha} wu \]

\[ C_D(\sin \alpha) v^2 \approx (C_{D_0} + C_{D_\alpha} \alpha^2) \frac{w}{u} u^2 \approx C_{D_0} wu + C_{D_\alpha} u^2 w^2 \]

But this term would approach \( \infty \) as \( u \to 0 \). Therefore instead of a parabolic drag approximation, two straight lines will be used as shown in Figure 10.
\[ C_D \sin \alpha u^2 = \left[ C_{D_0} + C_{D\alpha} \left( \frac{w}{u} - \alpha_1 \right) \right] \frac{w}{u} u^2 = C_{D_0} wu + C_{D\alpha} (w - u \alpha_1) w \]
valid only for \((w - u \alpha_1) > 0\)

\[ C_M w^2 = C_M \alpha u^2 + C_M \omega \dot{u} u^2 + C_M \varepsilon_c \varepsilon w u^2 \]
\[ = C_M \alpha wu + C_M \omega \dot{u} u^2 + C_M \varepsilon_c \varepsilon w \]

Using the results of these simplifications, the force equations may now be rearranged in programming form and written:

\[ -\ddot{u} = \frac{T_P}{M} \cos \sigma_p - \frac{T_L}{M} \cos \sigma_L + (\dot{M}_L + \dot{M}_P) \frac{u}{M} \]
\[ + C_D \frac{p_s}{2m} u^2 + (C_{D\alpha}^2 - C_{I\alpha}) \frac{p_s}{2m} w^2 \]
\[ + g \omega + \dot{\omega} w \]
\[ -\ddot{w} = \frac{T_P}{M} \sin \sigma_p - \frac{T_L}{M} \sin \sigma_L + (\dot{M}_L + \dot{M}_P) \frac{w}{M} \]
\[ + (C_{I\alpha} + C_{D_0}) \frac{p_s}{2m} w u + C_D \alpha \left( w - u \alpha_1 \right) \frac{p_s}{2m} w \]
\[ \text{only for } \]
\[ \text{values} \]
\[ -g - \frac{3}{15} \frac{M_L}{I_Y} \sin \sigma_L - \frac{T_P}{I_Y} \sin \sigma_p + 2 \frac{M_L}{I_Y} + 10 \frac{M_L}{I_Y} \]
\[ \dot{\theta} = 10 \frac{T_L}{I_Y} \sin \sigma_L - 15 \frac{T_P}{I_Y} \sin \sigma_p + 2 \frac{M_L}{I_Y} + 10 \frac{M_L}{I_Y} \]
\[ + C_{M\alpha} \frac{p_s c}{2I_Y} w u + (u C_{M\omega}) \frac{p_s c}{2I_Y} \dot{\omega} u + C_{M\varepsilon c} \varepsilon c \frac{p_s c}{2I_Y} u^2 \]

Now introducing the previously derived aerodynamic and force parameters, these equations may be written in computer programming form:

\[ -\ddot{u} = -1.48 \times 10^{-5} (\text{RPM}_P)^3 \left[ 1 + .29 \times 10^{-3} u \right] \cos \sigma_p \]
\[ -2.22 \times 10^{-5} (\text{RPM}_L)^3 \cos \sigma_L \]
\[ + 7.8 \times 10^{-5} (\text{RPM}_P) \left[ 1 + .18 \times 10^{-3} u \right] u + 11.8 \times 10^{-5} (\text{RPM}_L) u \]
\[ + .460 \times 10^{-5} u^2 + 32.2 \dot{\theta} + \dot{\omega} w \]
- \( w = 1.43 \times 10^{-5} (\text{RPM}_p)^3 \left[ 1 + .29 \times 10^{-3} u \right] \sin \sigma_p \)
+ \( 2.22 \times 10^{-5} (\text{RPM}_L)^3 \sin \sigma_L + 7.8 \times 10^{-5} (\text{RPM}_p) \left[ 1 + .18 \times 10^{-3} u \right] w \)
+ \( 11.8 \times 10^{-5} (\text{RPM}_L) w + 95 \times 10^{-5} wu + 28.4 \times 10^{-5} (w - .07u) w \)
- \( 32.2 - \epsilon_u \)

\[ + \frac{\partial \epsilon'}{\partial u} = - .222 \times 10^{-5} (\text{RPM}_p)^3 \left[ 1 + .29 \times 10^{-3} u \right] \sin \sigma_p \]
+ \( .222 \times 10^{-5} (\text{RPM}_L)^3 \sin \sigma_L + .236 \times 10^{-5} (\text{RPM}_L) u \)
+ \( 1.18 \times 10^{-5} (\text{RPM}_L) w - 1.79 \times 10^{-5} wu - 11.2 \times 10^{-5} \dot{u} u \)
+ \( 1.61 \times 10^{-5} \epsilon_c u^2 \)

Note that the \( w^2 \) term has dropped out of the drag equation because for this airplane \( C_{\text{D}a^2} = C_{l\alpha} \) within the accuracy of the available wind tunnel data.
APPENDIX IV
MODIFIED DESIGN EQUATIONS

Force Equations

Referring to the following sketch, the force equations for the modified design may be written.

\[ F_x = T_p + T_L \cos \alpha_L - M \dot{u} - C_D q S \cos \alpha + C_L q S \sin \alpha \]

\[ -m g \sin \theta - M(\dot{u} + \dot{\omega}) = 0 \]

\[ F_z = -T_L \sin \alpha_L - M \dot{w} - C_D q S \sin \alpha - C_L q S \cos \alpha \]

\[ + m g \cos \theta - M(\ddot{u} - \dot{\omega}) = 0 \]

\[ M = 2M_L u + (C_M \alpha + C_M \delta_c) q \dot{\sigma} + M_{REAC} - M_{\dot{\gamma}}^2 \dot{\gamma} = 0 \]

Since \( T_p \) has a fixed direction and is parallel to the inlet forces at \( \alpha = 0 \), \( T_p \) may now be treated as "net" thrust and the inlet momentum forces neglected. Therefore, using the new \( T_L \) equal to \( 5/3 \) of the old \( T_L \) we get:

\[ T_p = 0.011 \text{ RPM}_p^3 \]

\[ T_L = 5/3 (0.0165) (\text{RPM}_L)^3 = 0.0275 \text{ RPM}_L^3 \]

\[ M_L = 5/3 (0.0382) \text{ RPM}_L = 0.147 \text{ RPM}_L \]
Using the same procedure as in Appendix II to obtain the characteristic determinant for the hovering condition, we get:

\[ X_u = \frac{1}{M} (- \dot{M}_L) = -\frac{.147 \text{ RPM}_{Lw}}{745} = -0.019 \]

\[ X_w = 0 \]
\[ X_I = 0 \]
\[ Z_u = 0 \]
\[ Z_w = -\frac{\dot{M}_L}{M} = -0.019 \]
\[ Z_I = 0 \]
\[ M_u = 2 \frac{\dot{M}_L}{I_Y} = \frac{.147 \text{ RPM}_{Lw}}{7.45 \times 10^4} = +3.77 \times 10^{-4} \]
\[ M_w = 0 \]
\[ M_I = 0 \]

\[
\begin{vmatrix}
.019 + d & 0 & 32.2 \\
0 & (.019 + d) & 0 \\
+3.77 \times 10^{-4} & 0 & -d^2 \\
\end{vmatrix}
\]

\[ d^3 + .019 d^2 + .0121 = 0 \]

\[ \lambda = -0.23 \]
\[ \lambda = +0.10 \pm j0.20 \]

These roots are shown in Figure 28 and compared with the S-55 helicopter.
Comparison of the lift and drag polars for a typical winged and wingless configuration. $C_L$ and $C_D$ are based upon the same planform area of the body.
FIGURE 2
LIFT-DRAG CHARACTERISTICS FOR SEVERAL BODIES HAVING AN ELLIPTICAL CROSS SECTION AT M = 0.9
(VALUES FROM REFERENCES 2 AND 3)

REFERENCE AREA = $L^2$

ALL MODELS HAVE IDENTICAL CROSS SECTIONAL AREA

\[ \lambda = \frac{3}{b} \]
Slope of the lift curve variation and aerodynamic center movement with increasing mach number

Figure 3

Ref. 15
Ref. 10
Ref. 11
Effects of Fuselage cross section on static longitudinal
stability characteristics (Ref. 4)

Figure 4
Figure 5

Low Speed Wind Tunnel Results of a Wingless Jet VTOL
Figure 6a

The SHORT SC-1 Test Vehicle
Figure 6b
The Bell Air Test Vehicle
Rotating Turbojet Engine

Figure 6c
The Bell X-14
Deflected Exhaust Test Vehicle
FIGURE 7

ORIGINAL AIRCRAFT CONFIGURATION
FIGURE 8
LONGITUDINAL AERODYNAMIC STABILITY COEFFICIENTS
OF AIRCRAFT SIMULATED HEREIN.

--- --- --- VALUES FROM REFERENCE 12

APPROXIMATIONS USED

\[ C_M \quad C_D \quad C_L \]

-1.0 1.0

-0.8 0.32 - 0.8

-0.6 0.24 - 0.6

-0.4 0.16 - 0.4

-0.2 0.08 - 0.2

0 4 8 12 16

\[ \theta \]

\[ C_M (\delta_c = 0) \]

\[ C_D \]

\[ C_L \]

\[ (\text{PARABOLIC APPROX.}) \]

\[ (\text{LINEAR APPROX}) \]

\[ C_M (\delta_c = 9.7^\circ) \]
FIGURE 11

ROOT LOCUS AS VELOCITY VARIES, CALCULATED FROM SERIES OF STEADY STATE PERTURBATIONS.
ORIGINAL CONFIGURATION.

\[ M_0 = 400 \]

\[ 300 \]

\[ 200 \]

\[ 100 \]
FIGURE 12
BASIC ANALOG COMPUTER PROG.
<table>
<thead>
<tr>
<th>AIRCRAFT POTENTIOMETERS</th>
<th>POTENTIAL SETTINGS</th>
<th>PHYSICAL PARAMETER</th>
<th>CONTROL INPUT POTENTIOMETER SETTINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>POT. NO.</td>
<td>SETTING</td>
<td>POT. ID.</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>.10</td>
<td>1</td>
<td>.590</td>
</tr>
<tr>
<td>4</td>
<td>.172</td>
<td>2</td>
<td>.503</td>
</tr>
<tr>
<td>5</td>
<td>.296</td>
<td>19</td>
<td>2TP cos φ/</td>
</tr>
<tr>
<td>6</td>
<td>.39</td>
<td>20</td>
<td>2TP cos φ/</td>
</tr>
<tr>
<td>7</td>
<td>.73</td>
<td>21</td>
<td>2TP u/M</td>
</tr>
<tr>
<td>8</td>
<td>.39</td>
<td>22</td>
<td>2TP u/M</td>
</tr>
<tr>
<td>9</td>
<td>.161</td>
<td>23</td>
<td>2TP sin φ/</td>
</tr>
<tr>
<td>10</td>
<td>.230</td>
<td>24</td>
<td>2TP sin φ/</td>
</tr>
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<td>.258</td>
<td>25</td>
<td>2TP w/M</td>
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<td>12</td>
<td>.056</td>
<td>26</td>
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<tr>
<td>13</td>
<td>.70</td>
<td>27</td>
<td>4TP u/M</td>
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<tr>
<td>14</td>
<td>.446</td>
<td>29</td>
<td>1. e. + [w]</td>
</tr>
<tr>
<td>15</td>
<td>.10</td>
<td>33</td>
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<td>16</td>
<td>.475</td>
<td>34</td>
<td>1. e. [270]</td>
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<tr>
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<td></td>
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</tr>
<tr>
<td>θ₀ (rpm)</td>
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<td>60.3</td>
<td>300</td>
</tr>
<tr>
<td>w₀ (rpm)</td>
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<td>71.1</td>
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<tr>
<td>φ (rad.)</td>
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<td>.197</td>
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<td>REM (k)</td>
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<td>91</td>
<td>95</td>
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<tr>
<td>θ (deg.)</td>
<td>0</td>
<td>90</td>
<td>71</td>
</tr>
</tbody>
</table>
Figure 1b

Disturbance Response
\[ \mu_0 = 0 \quad \theta_0 = 0 \]

Original Design
Figure 15

Disturbance Response
$\lambda_0 = 100 \quad \theta_0 = 11$

Original Design

$\delta_c \rightarrow 1$ Sec.
Figure 16

Disturbance Response
\( u = 200 \), \( \theta = 0 \)
Original Design

\( u \)
300
200
100
0

\( \omega \)
40
30
20
10
0
-10
-20
-30

\( \theta \)
5
4
3
2
1
0
-1
-2
-3

\( \delta_c \)
5
4
3
2
1
0
-1
-2

\( u = 200 \), \( \theta = 0 \)

\[ \text{5 Sec.} \]
Figure 17

Disturbance Response

$u_0 = 200 \quad \theta_0 = 10$  
Original Design
Figure 16

Disturbance Response

$\mu_0 = 300$, $\theta_0 = 0$

Original Design

\[ \omega \]

\[ \theta \]

\[ \delta_c \]

$5 \text{ Sec.}$
Figure 19

Disturbance Response
\( U_0 = 300 \quad \theta_0 = 8.4^\circ \)

Original Design

---

\[ U_0 = 300 \quad \theta_0 = 8.4^\circ \times 1.47 \text{ RAD} \]

5 Sec.
Figure 23

Disturbance Response

\[ \mu_0 = 400, \ \phi_0 = 13^\circ \]

original design

\[ \mu = \text{variable}, \ \phi = \text{variable} \]

\[ \Delta = \text{variable}, \ \delta = \text{variable} \]

\[ \Delta = 420, \ \phi = 12.5^\circ \]

\[ \text{Sec.} \]
FIGURE 23
SIMULATOR CONNECTIONS
FOR ORIGINAL DESIGN
Figure 2a
Disturbance Response
$H_0 = 250 \quad \theta = 8^\circ$

Original Design with Simulator Inputs

\[ \begin{align*}
\delta_0 & = 0 \\
\mu & = 400 \\
\phi & = 200 \\
\theta & = 11 \\
\omega & = 0 \\
\end{align*} \]

5 Sec.
FIGURE 25
SIMULATOR FLIGHT - FORWARD TRANSITION
AND RETURN TO 300 KIAS, INCLUDING
CUTBACK OF LIFTING ENGINES: ORIGINAL DESIGN
Figure 26
Simulator Flight - Slowing Transition Attempt - Original Design
Figure 27

Final Aircraft Design
COMPARISON OF HOVERING ROOTS
OF 2 DESIGNS IN THIS REPORT AND
THE S-55 HELICOPTER

○ - ORIGINAL DESIGN
● - MODIFIED DESIGN
□ - S-55 HELICOPTER
FIGURE 29
SIMULATOR CONNECTIONS
FOR MODIFIED AIRCRAFT
Figure 30
Disturbance Response
\( U_0 = 0 \)

Final Design

\( U \)

\( \theta \)

\( ALT \)

200
100
0
-100
-10
-100

10
5
0
-10

5000
1000
3000
2000
1000
0

2 Sec.
Figure 3.
Disturbance Response
$\mu_0 = 60$

Final Design
Figure 32
Disturbance Response
\[ \mu_0 = 200 \]

Final Design
Final Design

Figure 13
Disturbance Response
\[ u_0 = 300 \]

\[ e_c \]

\[ u \]

\[ \Theta \]

\[ \text{ALT} \]

\[ \rightarrow \text{ } 5 \text{ sec.} \]
FIGURE 34

SIMULATOR FLIGHT - HOVERING TO 100 FPS AT 1000 FT AND RETURN - FINAL DESIGN
FIGURE 36
SIMULATOR FLIGHT - HOVERING TO 500 FPS AND RETURN WITH INTERMEDIATE TRIMMED CONDITIONS - FINAL DESIGN
A preliminary design and longitudinal dy